*Journal of Applied Mechanics and Technical Physics, Vol. 49, No. 1, pp. 105–108, 2008*

# EVALUATING ROCK MASS FAILURE IN THE VICINITY OF SLOT CUTS

**N. V. Cherdantsev, V. T. Presler, and V. Yu. Izakson**

UDC 622.241.54

*A comparative analysis of the failure of rock with surfaces of weakness in the vicinity of elongated clot cuts of rectangular cross section was performed using the boundary integral equation method and the Mohr–Kuznetsov strength criterion. The rock failure coefficient was used as the criterion of breaking.*

**Key words:** *stress state, slots in rock, discontinuity zone, rock failure coefficient.*

Slots of various cross sections are used to unload rock in the vicinity of mine tunnels. Elongated rectangular cuts (slots) in close proximity to a mine tunnel [1] are also used for degassing coal beds. To obtain quantitative and qualitative estimates of the degree of unloading, it is necessary to calculate the stability of the rock mass, i.e., to determine the stress field in the vicinity of the slot and to establish the regions in which the strength criteria are not satisfied.

We assume that the rock mass possesses strength anisotropy [2], i.e., it has ordered regular surfaces of weakness, whose strength characteristics are much lower than the strength characteristics of the main rock. Even in the stage of elasticity, this medium fails primarily along these surfaces to form discontinuity zones. According to the Mohr–Kuznetsov strength theory, the strength condition is formulated as follows:

$$\tau_n \leqslant \sigma_n n + K. \tag{1}$$

Here $\tau_n$ and $\sigma_n$ are the tangential and normal stresses on the surface of weakness, respectively, and $n$ and $K$ are the coefficients of internal friction and coupling of the surfaces of weakness. We also assume that rock mass failure along the surfaces of weakness does not lead to a stress redistribution.

Because the slot is elongated, i.e., its length $2l$ far exceeds the length of the cross-sectional contour, it follows that, in the vicinity of the slot, except in small areas in the vicinity of the end cross sections, rock is in a plane stress state. Therefore, a plane elastic problem is solved. We assume that vertical stresses $\sigma_{33}^{\infty} = \gamma H$ and horizontal stresses $\sigma_{11}^{\infty} = \sigma_{22}^{\infty} = \lambda\gamma H$ act at infinity, where $\lambda$ is the lateral pressure coefficient, $\gamma$ is the rock density, and $H$ is the depth of location of the slot (Fig. 1). The problem is solved using the boundary element method, which is employed in some problems of geomechanics, for example, in calculations of rock mass at the junction of two mine tunnels of square cross section [3]. In this method, the problem is reduced to solving the boundary integral equation of the second external boundary-value problem of elasticity theory [4–6]:

$$\frac{1}{2}\, a_q(Q_O) - \int\limits_L \Gamma_{qm}(Q_O, M_O) a_m(M_O)\, dL_{M_O} = n_q(Q_O)\sigma_{qq}^{\infty} - F_q(Q_O). \tag{2}$$

The influence tensor $\Gamma_{qm}(Q_O, M_O)$ is defined as

$$\Gamma_{qm} = \frac{1}{4\pi(1-\nu)r^2}\Big[(1-2\nu)(x_q n_m - x_m n_q) + \Big((1-2\nu)\delta_{qm} + 2\,\frac{x_q x_m}{r^2}\Big)\frac{x_t n_t}{r}\Big].$$

Institute of Coal and Coal Chemistry, Siberian Division, Russian Academy of Sciences, Kemerovo 650610; v.izaxon@kemsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 49, No. 1, pp. 129–133, January–February, 2008. Original article submitted August 31, 2006; revision submitted February 27, 2007.

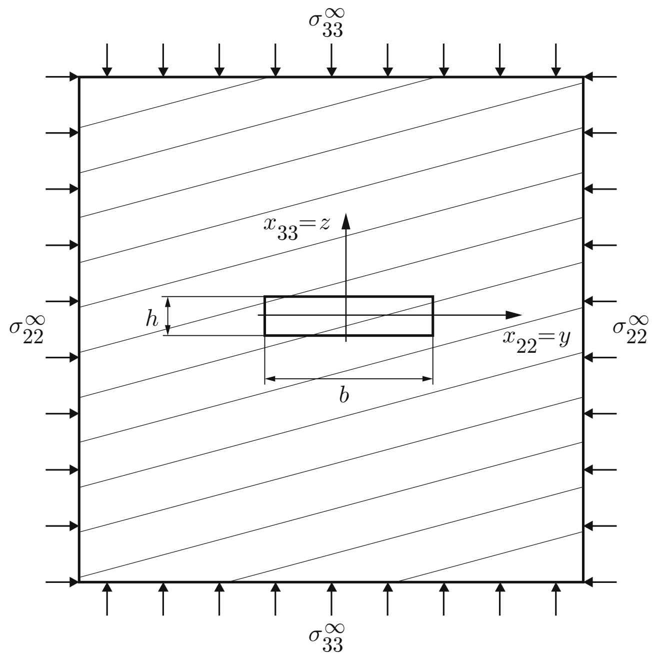



Fig. 1. Computation scheme of the problem: the inclined lines are the surfaces of weakness.

Here $\nu$ is Poisson ratio, $Q_O$, $M_O$, and $r$ are points on the slot studied and the distance between them, respectively, $\delta_{qm}$ is Kronecker's symbol, $\sigma_{qq}^{\infty}$ is the stress tensor at infinity, $L$ is the cross-sectional contour of the slot, $n_q$ and $n_m$ are the normal direction cosines to the slot contour at the points $Q_O$ and $M_O$, $F_q(Q_O)$ is the support response vector if a support is established, and $\boldsymbol{a}$ is the fictitious load vector; the subscripts $q$, $m$, and $t$ take values 1, 2, and 3.

Equation (2) is solved numerically. First, the slot contour is replaced by a finite number $N$ of linear elements, and the integral is replaced by a sum. Then, integration is performed over each element, assuming that the intensities $\boldsymbol{a}$ and $\boldsymbol{F}$ are constant within an element. As a result, we have

$$\frac{1}{2}\,(a_q^*)_i - \sum_{\substack{j=1\\ j\neq i}}^{N} (\Gamma_{qm})_{ij}(a_m^*)_j \Delta L_i = (n_q)_i (t_{qq}^{\infty})_i - (F_q^*)_i, \tag{3}$$

where the subscript $i$ corresponds to the number of the point on the slot contour at which the boundary condition is formulated, the subscript $j$ to the number of the current point on the contour; the summation is performed over all points of the contour, except for $j = i$; the superscript "$*$" corresponds to the resultant forces applied at the centers of the boundary elements:

$$(a_q^*)_i = (a_q)_i \Delta L_i, \qquad (a_m^*)_j = (a_m)_j \Delta L_j, \qquad (t_{qq}^{\infty})_i = (\sigma_{qq}^{\infty})_i \Delta L_i, \qquad (F_q^*)_i = (F_q)_i \Delta L_i.$$

Once Eqs. (3) is solved for $(a_q^*)_j$ at any point $i$ of the computation region of the rock mass, the stress tensor $\sigma_{qm}$ is determined using the superposition principle:

$$(\sigma_{qm})_i = (\sigma_{qmt})_{ij}(a_t^*)_j + (\sigma_{qq}^{\infty})_i. \tag{4}$$

In (4), $\sigma_{qmt}$ is the result of integration of the Kelvin solution (Kelvin tensor) over $x_1$ from $-l$ to $+l$ in the problem of the action of a unit force on an elastic space for $l \to \infty$ [6]. As a result, the solution of the plane problem of the action of the force becomes

$$\sigma_{qmt} = \frac{1}{4\pi(1-\nu)r^2}\Big((1-2\nu)(\delta_{mt}x_q + \delta_{qt}x_m - \delta_{qm}x_t) + \frac{2x_q x_m x_t}{r^2}\Big).$$

In the vicinity of the slot, the fractured regions, or discontinuity zones, also called the regions of instability of the rock mass, are found as a set of points, at which, according to the strength criterion (1), rock mass failed

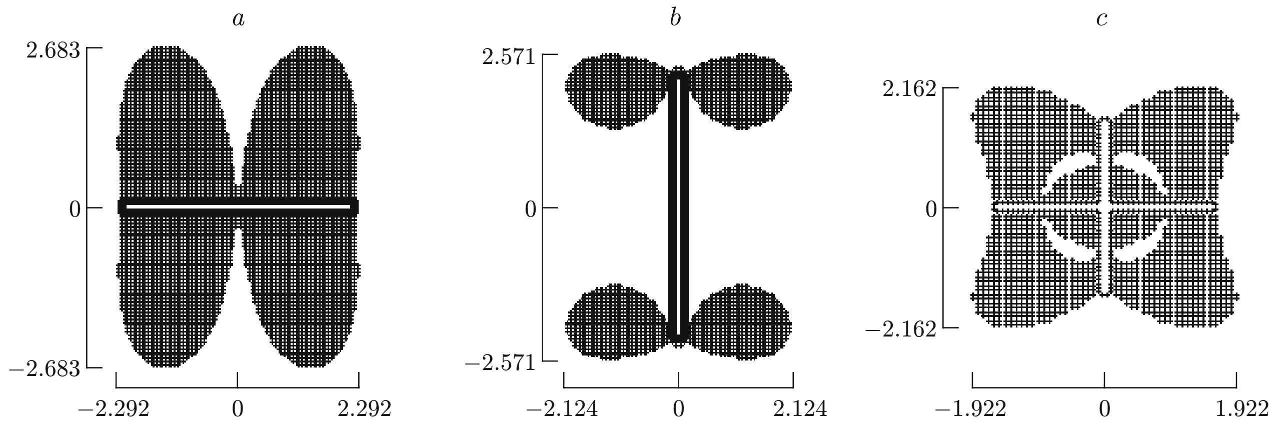
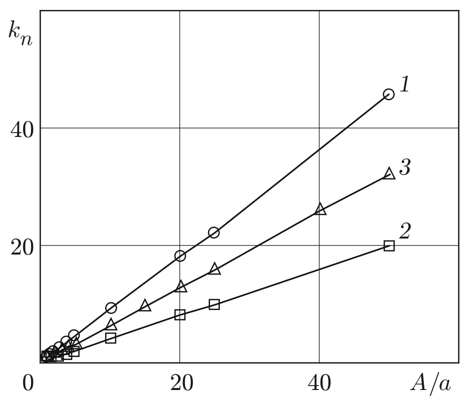



Fig. 2. Discontinuity zones in the vicinity of horizontal (a), vertical (b), and cruciform slots (c) for $A/a = 20$.



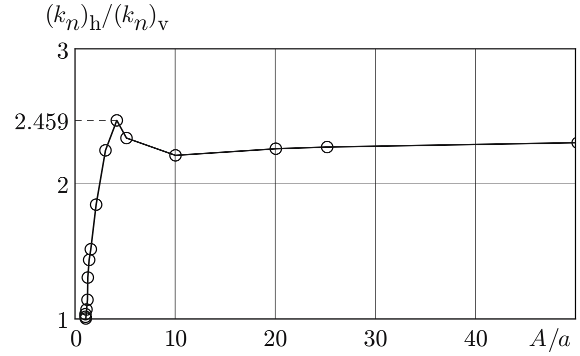




Fig. 3

Fig. 4

Fig. 3. Rock failure coefficient versus ratio of the characteristic cross-sectional dimensions $A/a$: 1) horizontal slot; 2) vertical slot; 3) cruciform slot.

Fig. 4. Rock failure coefficients for horizontal slot $(k_n)_\mathrm{h}$ and vertical slot $(k_n)_\mathrm{v}$ versus ratio of their characteristic dimensions.

along the surfaces of weakness. The degree of rock mass failure in the vicinity of the slot (the failure coefficient $k_n$) is defined as the ratio of the volume of the discontinuity zone to the volume of the slot (for an elongated slot, this ratio can be replaced by the ratio of the corresponding areas). In the case of a regular grid of the computation region,

$$k_n = N_n/N_c,$$

where $N_n$ is the number of fractured nodes in the computation region and $N_c$ is the number of nodes of the computation region which can be arranged within the rectangular slot.

Below, we give the results of a numerical experiment for slots of rectangular and cruciform cross sections, each of which has unit area. It is assumed that the stress field is hydrostatic ($\lambda = 1$), the surfaces of weakness are horizontal, the coupling coefficient $K = 0$, and the internal friction angle $\varphi = 20°$.

Figure 2a and b shows discontinuity zones in the vicinity of slots of rectangular cross section for $A/a = 20$ ($A$ is the dimension of the larger side and $a$ is the dimension of the smaller side). Discontinuity zones in the vicinity of an elongated slot of cruciform cross section (Fig. 2c) are constructed for the first time. This slot is a combination of horizontal and vertical slots. In Fig. 2 it is evident that in the case of a horizontal rectangular slot, the discontinuity zones are located along its larger side, and in the case of a vertical slot, the discontinuity zones are concentrated in the vicinity of its end cross sections.

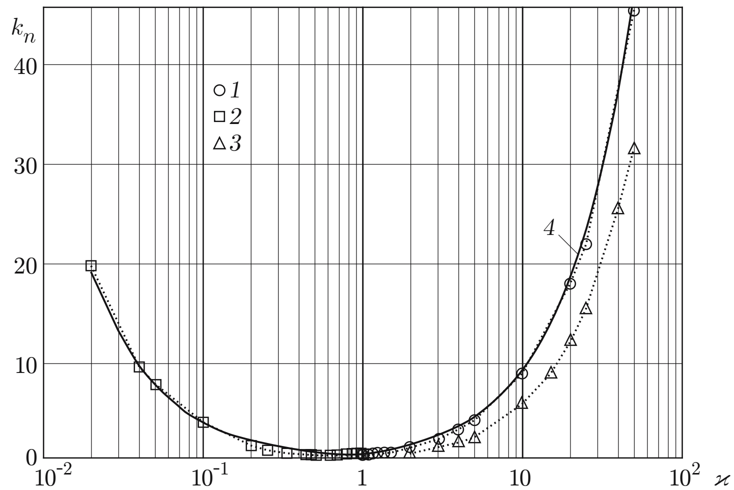



Fig. 5. Rock failure coefficient versus $\varkappa$ (the scale on the axis is logarithmic): 1) horizontal slot; 2) vertical slot; 3) cruciform slot; 4) approximating dependence.

Figure 3 shows the rock failure coefficient $k_n$ versus the characteristic dimensions of slot cuts. It is evident that these dependences are almost linear and that, in the vicinity of the horizontal slot (curve 1), the rock mass failure is larger than in the vicinity of the vertical slot (curve 2).

Figure 4 shows the ratio of the rock failure coefficients for horizontal slot $(k_n)_{\rm h}$ and vertical slot $(k_n)_{\rm v}$ versus ratio of their characteristic dimensions. It is evident that for $A/a > 10$, this ratio tends monotonically to the maximum value $(k_n)_{\rm h}/(k_n)_{\rm v} = 2.459$ obtained for $A/a = 4$.

Figure 5 shows the rock failure coefficient versus $\varkappa = b/h$ ($b$ is the horizontal dimension of the slot and $h$ is the vertical dimension). The curves of the rock failure coefficients for slots are well approximated by the dependence

$$k_n(\varkappa) = \begin{cases} 0.45 + 0.375\varkappa^{-1}, & \varkappa < 1, \\ 1.412 \cdot 10^{-4}\varkappa^2 + 0.916\varkappa + 0.142, & \varkappa > 1. \end{cases} \qquad (5)$$

For $\varkappa > 1$, curve (5) adequately curve 1, and for $\varkappa < 1$, it well approximates curve 2. From the analysis of the curves it follows that:

1. The rock failure coefficient is larger for horizontal slots than for vertical slots. The maximum ratio of the rock failure coefficients for horizontal and vertical slots $(k_n)_{\rm h}/(k_n)_{\rm v} = 2.459$ is reached for $A/a = 4$.

2. The smallest value of the rock failure coefficient $k_n = 0.915$ is obtained in the vicinity of a vertical slot for $A/a = 1.5$ (for a square slot, $k_n = 1.077$).

3. The rock failure coefficient for cruciform slots have intermediate values between the values of the coefficients for horizontal and vertical slots and is close to their arithmetic-mean values.

## REFERENCES


1. A. F. Éller, V. F. Gorbunov, and V. V. Aksenov, *Screw-Drive Shaft-Sinking Apparatus* [in Russian], Nauka, Novosibirsk (1992).
2. Zh. Erzhanov, V. Yu. Izakson, and V. M. Stankus, *Mole Excavations in Kuzbass Mines. Experience with Maintenance and Stability Computation* [in Russian], Kemerov. Knizh. Izd., Kemerovo (1976).
3. N. V. Cherdantsev and S. V. Cherdantsev, "Discontinuity zones in the junction region of two mine tunnels," *J. Appl. Mech. Tech. Phys.*, **45**, No. 4, 572–574 (2004).
4. N. V. Cherdantsev and V. A. Shalamanov, "Boundary integral equations in problems of the mechanics of underground structure," *Izv. Vyssh. Uchebn. Zaved., Gorn. Zh.*, No. 5, 50–54 (2004).
5. N. V. Cherdantsev and V. Yu. Izakson, *Some Three- and Two-Dimensional Problems of Geomechanics* [in Russian], Uzd. Kuzbass. Gos. Univ., Kemerovo (2004).
6. T. Cruse and F. Rizzo (eds.), *Boundary Integral Equation Method: Computational Applications in Applied Mechanics*, ASME, New York (1975).